DEFLECTION OF A JET INJECTED INTO A STREAM

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A solution is given for the deflection of the axis of two-dimensional and circular jets by an entraining stream. Design formulas are obtained and compared with experiment.

The problem of determining the shape of a jet injected into an entraining stream is part of the general problem of the mixing of streams of gas, which owing to its great practical interest is currently receiving the attention of many researchers.

Suppose a jet of gas with the initial parameters ρ_{v0}, v_0 is injected at an angle α_0 to the axis through an aperture in the side wall of a channel in which another gas with density ρ_w is flowing at the uniform velocity w. The injected jet will be deflected by the stream. It is required to find the curve followed by its axis.

The analytical solutions of this problem given in the literature may all be divided into two groups. The first group contains the purely kinematic solutions of [1-4], in which some form of the method of geometric superposition of the jet and the stream is used. However, although these solutions are valid for ideal fluids and potential flow, their application to real fluids can be regarded only as a coarse approximation, and therefore the results obtained show poor agreement with experiment, as pointed out in [5].

The second group of analytic solutions of the problem contains the dynamic solutions of [6-8], based on an examination of the forces exerted on the jet by the stream. In [7] the jet is regarded as an airfoil on which the aerodynamic force of the stream acts. Equating the normal component of the aerodynamic force to the centrifugal force, the author calculates the radius of curvature at each point of the trajectory and then finds the equation of the axis of the jet, which, in the particular case of a circular jet with $\alpha_0 = \pi/2$, takes the form:

$$\frac{y}{d_0} = 14.4 \sqrt{\frac{\rho_{v_0} v_0^2}{c_n \rho_w w^2}} \lg\left[1 + 0.1 \frac{x}{d_0}\left(1 + \sqrt{1 + 20\frac{d_0}{x}}\right)\right]. \tag{1}$$

It is known from experimental aerodynamics that the value c_n is of the order of unity. The author recommends using $c_n = 3$. However, formula (1) then gives a large discrepancy with experiment (up to 300%). To make this formula agree with the experimental airfoil drag data, c_n must be increased by a factor of 25-30, i. e., to values that would be hard to account for theoretically.

In this paper an attempt is made to find the shape of the jet axis in the stream from the value of the drag represented by the jet. Let us first consider an infinite two-dimensional jet (Fig. 1). Since a free jet is isobaric, the projection of its momentum on the y axis remains unchanged:

$$\rho_v v^2 S_n \sin\alpha = \rho_{v_0} v_0^2 S_{n_0} \sin\alpha_0. \tag{2}$$

Hence we obtain the first equation of motion of a jet issuing from an infinite slot:

$$\frac{dy}{dt} = v\sin\alpha = \rho_{v_0} v_0^2 S_{n_0} \sin\alpha_0 (\rho_v v S_n)^{-1}. \tag{3}$$

Let us consider a point M on the jet axis with coordinates x, y and calculate the pressure of the stream on a section of the jet from 0 to M per unit width of the jet. Let us assume that the aerodynamic force P of the flow acts only along the x axis and is proportional to the velocity head:

$$P = \frac{1}{2} c_x y \rho_w w^2. \tag{4}$$

The law of change of momentum takes the form:

$$\rho_v v^2 S_n \cos\alpha - \rho_{v_0} v_0^2 S_{n_0} \cos\alpha_0 = P, \tag{5}$$

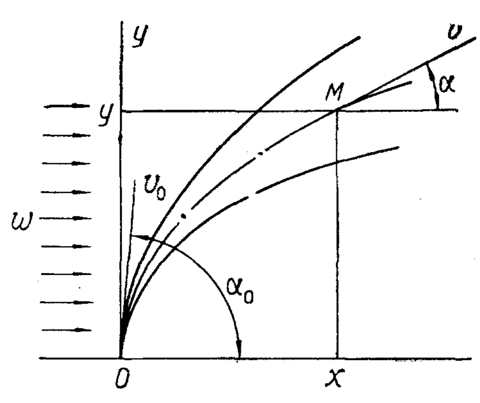

Fig. 1. Diagram of jet deflected by a flow of gas.

from which we obtain the second equation of motion of the jet:

$$\frac{dx}{dt} = v\cos\alpha = \frac{1}{2}\left(c_x y\,\rho_w w^2 + 2\rho_{v_0} v_0^2 S_{n_0}\cos\alpha_0\right)(\rho_v v S_n)^{-1}. \tag{6}$$

Dividing (6) by (3), we obtain the differential equation of the jet axis

$$\frac{dx}{dy} = \frac{1}{2}c_x y\,\rho_w w^2 (S_{n_0}\sin\alpha_0 \rho_{v_0} v_0^2)^{-1} + \operatorname{ctg}\alpha_0, \tag{7}$$

where $S_{n_0} = b_0 \cdot 1$. The equation of the axis of the deflected jet is found by integrating (7) with the boundary conditions $x|_{y=0} = 0$:

$$\frac{x}{b_0} = \frac{1}{4}c_x\,\rho_w w^2 (\rho_{v_0} v_0^2 \sin\alpha_0)^{-1}\left(\frac{y}{b_0}\right)^2 + \frac{y}{b_0}\operatorname{ctg}\alpha_0. \tag{8}$$

In Fig. 2, (8), with $c_x = 5$ and $\alpha_0 = \pi/2$, is compared with the experimental data of [5] for a two-dimensional jet. An examination of Fig. 2 shows that, for $\frac{\rho_w w^2}{\rho_{v_0} v_0^2} = 0.0025$ and 0.08, the experimental data fall sufficiently close to the theoretical curve. Some deviation (up to 20%) is observed for $\frac{\rho_w w^2}{\rho_{v_0} v_0^2} = 0.01$.

Let us now proceed to solve the problem of the deflection of the axis of a jet that is circular in section. It has been established [7] from an analysis of experimental data that, under the action of the stream, the jet section becomes distorted even at a small distance ($l/d_0 \approx 1.5$) from the nozzle, acquiring a horseshoe shape with a ratio of the sides ϑ/h of the order of 1:5. The width of the horseshoe increases approximately in proportion to the distance from the nozzle:

$$h = h_0 + c_1 l, \tag{9}$$

where h_0 is the nominal width of the initial section. We shall assume, as in [7], that the initial jet section is an ellipse equal in size to a circle of the initial diameter ($\delta_0 h_0 = d_0^2$) and that for $h_0 = 5\delta_0$, $\delta_0 = 0.45d_0$, $h = 2.25d_0$, whence $S_{n_0} = \frac{1}{4}\pi d_0^2$. Assuming that the coefficient of angular expansion of the deflected jet of horseshoe section is the same as that for a straight jet $c_1 = 0.22$, we obtain

$$h = 2.25\,d_0 + 0.22\,l. \tag{10}$$

Consider a point with coordinates x, y on the axis of the jet. From the condition that the projection of the jet momentum

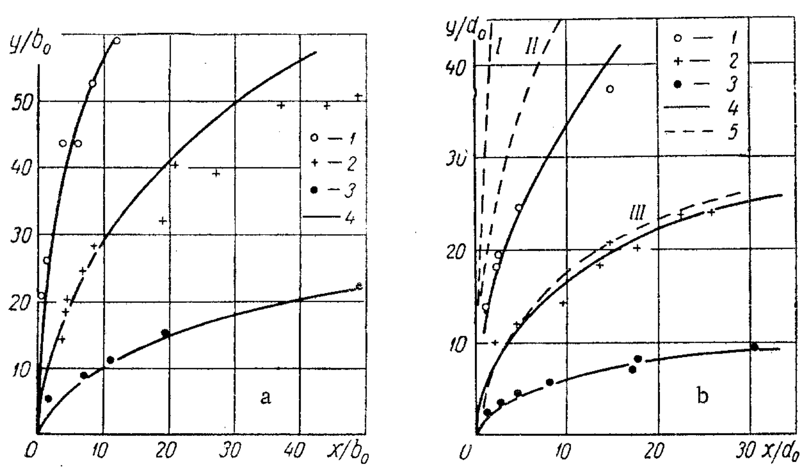

Fig. 2. Comparison with the experimental data of [5] for a two-dimensional jet: (a) 4 – according to (8) and for a circular jet, (b) 4 – according to (20), 5 – according to (1) with $c_n = 3$ (I, II, and III correspond to velocity head ratios of 0.0025, 0.01 and 0.08): 1 – $\rho_w w^2/\rho_{v_0} v_0^2 = 0.0025$; 2 – 0.01; 3 – 0.08

on the y axis is constant, we have

$$\frac{dy}{dt} = v \sin \alpha = \rho_{v_0} v_0^2 S_{n_0} \sin \alpha_0 (\rho_v v S_n)^{-1}. \tag{11}$$

Since we are assuming that the aerodynamic flow force P acts only along the x axis,

$$P = \frac{1}{2} c_x \rho_w w^2 F. \tag{12}$$

This projection has approximately the shape of an equilateral trapezium with sides h_0 and h and height y. Therefore $F = \frac{1}{2}(h_0 + h)\, y$ or, taking (10) into account,

$$F = \frac{1}{2} y (2h_0 + c_1 l). \tag{13}$$

Substituting the value of P found from (12) in (5), we obtain the second equation of motion:

$$\frac{dx}{dt} = v \cos \alpha = \frac{1}{4} [c_x y (2h_0 + c_1 l) \rho_w w^2 + 4\rho_{v_0} v_0^2 S_{n_0} \cos \alpha_0] (S_n \rho_v v)^{-1}. \tag{14}$$

Dividing (14) by (11), we obtain the differential equation of the axis of a jet issuing from a circular aperture:

$$\frac{dx}{dy} = c_x y (2h_0 + c_1 l) \rho_w w^2 (4\rho_{v_0} v_0^2 S_{n_0} \sin \alpha_0)^{-1} + \operatorname{ctg} \alpha_0. \tag{15}$$

The solution of (15) may be obtained by the method of successive approximations. We can take the equation of a two-dimensional jet $y_0(x)$ as the first approximation, then find $l_0 = \int_0^x \sqrt{1 + y_0'^2}\, dx$ and, substituting this value for l_0 b in (15), find the second approximation $y_1(x)$ for the jet axis. After this, we again determine $l_1 = \int_0^x \sqrt{1 + y_1'^2}\, dx$. Substituting the value found for l_1 in (15), we find $y_2(x)$ in the third approximation, and so on. Because of the complexity of the function $l(x)$, the solution described is too cumbersome. A coarser approximation may be obtained if we assume that the width of the horseshoe shaped section of the deflected jet is proportional not to the distance l from the nozzle, but to the coordinate x. This assumption obviously gives a larger error for the initial part of the jet, where for $\alpha_0 = \pi/2$ $l \gg x$, and in general, it is invalid for jets with $\alpha_0 > \pi/2$.

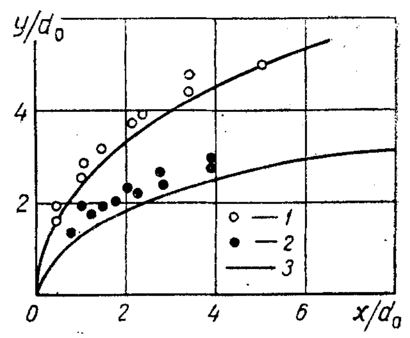

Fig. 3. Comparison with the experimental data of [9] for circular jets: 1 – $\rho_W w^2/\rho_{v_0} v_0^2 = 0.061$; 2 – 0.21; 3 – according to (20)

Then, instead of (10), we shall have

$$h = 2.25\, d_0 + 0.22\, x \tag{16}$$

and instead of (15):

$$\frac{dx}{dy} = c_x y \rho_w w^2 (4\rho_{v_0} v_0^2 S_{n_0} \sin \alpha_0)^{-1} (2h_0 + c_1 x) + \operatorname{ctg} \alpha_0. \tag{17}$$

This linear inhomogeneous equation may be integrated by the method of variation of arbitrary constants, and with boundary conditions $x|_{y=0} = 0$ leads to the expression:

$$\ln \left| 1 + \frac{c_1 x}{2h_0} \right| = c_x c_1 \rho_w w^2 y^2 (8\rho_{v_0} v_0^2 S_{n_0} \sin \alpha_0)^{-1} + \ln \left| 1 + c_1 \operatorname{ctg} \alpha_0 \sqrt{2\pi}\, \Phi \left[\left(\frac{c_1 c_x \rho_w w^2 y}{4\rho_{v_0} v_0^2 S_{n_0} \sin \alpha_0} \right)^{\frac{1}{2}} \right] \right|, \tag{18}$$

where $\Phi(s) = \frac{1}{\sqrt{2\pi}} \int_0^s \left[\exp\left(-\frac{t^2}{2} \right) \right] dt$ is the Laplace function.

If we substitute numerical values of all the coefficients in (18), then, for the case of a jet injected at a right angle to the flow ($\alpha_0 = \pi/2$), we obtain the following result:

$$\lg\left|1+0.049\frac{x}{d_0}\right| = 0.0153\, c_x\, \rho_w w^2 (\rho_{v_0} v_0^2)^{-1}\left(\frac{y}{d_0}\right)^2, \tag{19}$$

whence

$$\frac{y}{d_0} = 16.2\left[\rho_{v_0} v_0^2 (\rho_w w^2 c_x)^{-1} \lg\left(1+0.049\frac{x}{d_0}\right)\right]^{\frac{1}{2}}. \tag{20}$$

Curves constructed from (20) with $c_x = 4$ and the experimental data of [5] and [9] are compared in Fig. 2b and Fig. 3. It can be seen that the theoretical curves fall close to the experimental data, the deviation not exceeding ±20%, which may be considered quite satisfactory.

Notation

b – width of two-dimensional jet; c_1 – coefficient of angular expansion of jet; c_n – resistance coefficient of airfoil for normal velocity component; c_x – drag coefficient of jet in relation to stream; d – diameter of circular jet; F – projected area of jet on a plane perpendicular to flow; h – width of a jet of horseshoe section; l – length of jet from nozzle; P – aerodynamic flow force; S_n – normal section of jet; v, v_x, v_y – gas velocity in the jet and its projection on the x and y axes; w – velocity of stream; α – angle between mean gas velocity in jet and velocity of stream; δ – thickness of horseshoe section of jet; ρ_v, ρ_w – gas density in jet and stream; 0 – subscript referring to initial section.

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